Moment Invariants
for Definiens Developer 7.0
Short Reference

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This software plug-in calculates various known moment invariants as object features within Definiens Developer. It has been developed with the Definiens SDK[1] and the IT++ [2] library.

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1 Installation

Copy the `DIAPropDscrMoments.dll` to Definiens Plug-in Directory, by default this is located at `'C:\Program Files\Definiens Developer 7.0\bin\plugins'`.

Now run Definiens and the Moments Invariants will be listed as object features (Fig. 1).

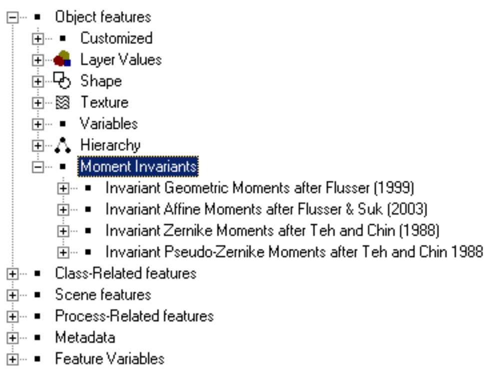

Fig. 1: Moment invariants in Definiens Developer

[*] Implementation
[1] http://www.definiens.com/definiens-sdk_131_7_8.html
[2] http://itpp.sourceforge.net/

2 Usage

Every invariant feature has to be chosen manual by "`Create new 'Invariant [...]'`", where the parameters have to be edited. The `Layer` gives the image density function $f(x, y)$ for the moment function, where the other parameters specifies the properties of the invariant. There are four types of moment invariants implemented: Geometric (GMI), Affine (AMI), Zernike (ZMI) and Pseudo-Zernike (PZMI). For a more detailed introduction and review of moment invariants please see Kristinsdóttir (2008).

2.1 Moment Invariants

For Cartesian coordinates a $(p+q)$ order moment generating function Φ_{pq} is defined by

$$\Phi_{pq} = \int\int \Psi_{pq}(x, y) f(x, y) \mathrm{d}x \mathrm{d}y \tag{1}$$

where Ψ_{pq} denotes a kernel function. Most common after Hu (1962) the monomial

$$\Psi_{pq}(x, y) = x^p y^q \tag{2}$$

is chosen.

However, digital image functions are commonly discrete, thus we follow the suggestions of Flusser (2000a) to achieve an exact integration

$$m_{pq} = \frac{1}{(p+1)(q+1)} \sum_{i=1}^{n} \sum_{j=1}^{m} \left((i + 1/2)^{p+1} - (i - 1/2)^{p+1} \right) \left((j + 1/2)^{q+1} - (j - 1/2)^{q+1} \right) f(i, j) \tag{3}$$

Moments of order $p + q > 1$ are centralized by the intensity barycenter of given image function

$$x_c = \frac{m_{10}}{m_{00}},\ y_c = \frac{m_{01}}{m_{00}} \tag{4}$$

Hence m_{pq} becomes a central moment, which is invariant to translation

$$\mu_{pq} = m_{pq}(x - x_c, y - y_c) \tag{5}$$

and according to achieve scale invariance μ_{pq} is scaled

$$\eta_{pq} = \frac{\mu_{pq}}{\mu_{oo}^{1/2(p+q+2)}} \tag{6}$$

2.1.1 Geometric Moments Invariants

The following set of six independent invariants with second and third order moments after Flusser (2000b) are available:

$$\phi_1 = \eta_{20} + \eta_{02} \tag{7}$$

$$\phi_2 = (\eta_{30} + \eta_{12})^2 + (\eta_{21} + \eta_{03})^2 \tag{8}$$

$$\phi_3 = (\eta_{20} - \eta_{02}) \left((\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2 \right) + 4\eta_{11}(\eta_{30} - \eta_{12})(\eta_{21} + \eta_{03}) \tag{9}$$

$$\phi_4 = \eta_{11} \left((\eta_{30} + \eta_{12})^2 - (\eta_{03} + \eta_{21})^2 \right) - (\eta_{20} - \eta_{02})(\eta_{03} + \eta_{12})(\eta_{21} + \eta_{03}) \tag{10}$$

$$\phi_5 = (\eta_{30} - 3\eta_{12})(\eta_{30} + \eta_{12}) \left((\eta_{30} - \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2 \right) + (3\eta_{21} - \eta_{03})(\eta_{21} + \eta_{03}) \left(3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2 \right) \tag{11}$$

$$\phi_6 = (3\eta_{21} - \eta_{03})(\eta_{30} + \eta_{12}) \left((\eta_{30} + \eta_{12})^2 - 3(\eta_{21} + \eta_{03})^2 \right) - (\eta_{30} - 3\eta_{12})(\eta_{21} + \eta_{03}) \left(3(\eta_{30} + \eta_{12})^2 - (\eta_{21} + \eta_{03})^2 \right) \tag{12}$$

2.1.2 Affine Moment Invariants

Suk and Flusser (2003) presented six affine moment invariants of the third, fifth and seventh orders. We have implemented the following five invariants:

$$
\begin{aligned}
I_1 &= (\mu_{30}^2\mu_{03}^2 - 6\mu_{30}\mu_{32}\mu_{12}\mu_{03} + 4\mu_{30}\mu_{12}^3 + 4\mu_{21}^3\mu_{03} - 3\mu_{21}^2\mu_{12}^2)/\mu_{00}^{10} && (13)\\
I_2 &= (\mu_{50}^2\mu_{05}^2 - 10\mu_{50}\mu_{41}\mu_{14}\mu_{05} + 4\mu_{50}\mu_{32}\mu_{23}\mu_{05} + 16\mu_{50}\mu_{32}\mu_{14}^2 - 12\mu_{50}\mu_{23}^2\mu_{14} \\
&\quad +16\mu_{41}^2\mu_{23}\mu_{05} + 9\mu_{41}^2\mu_{14}^2 - 12\mu_{41}\mu_{32}^2\mu_{05} - 76\mu_{41}\mu_{32}\mu_{32}\mu_{14} + 48\mu_{41}\mu_{23}^2 \\
&\quad +48\mu_{32}^3\mu_{14} - 32\mu_{32}^2\mu_{23}^2)/\mu_{00}^{14} && (14)\\
I_3 &= (\mu_{30}^2\mu_{12}\mu_{05} - \mu_{30}^2\mu_{03}\mu_{14} - \mu_{30}\mu_{12}^2\mu_{05} - 2\mu_{30}\mu_{21}\mu_{12}\mu_{14} + 4\mu_{30}\mu_{21}\mu_{03}\mu_{23} + 2\mu_{30}\mu_{12}^2\mu_{23} \\
&\quad -4\mu_{30}\mu_{12}\mu_{03}\mu_{32} + \mu_{30}\mu_{03}^2\mu_{14} + 3\mu_{21}^3\mu_{14} - 6\mu_{21}^2\mu_{12}\mu_{23} - 2\mu_{21}^2\mu_{03}\mu_{32} + 6\mu_{21}^2\mu_{12}\mu_{32} \\
&\quad +2\mu_{21}\mu_{12}\mu_{30}\mu_{41} - \mu_{21}\mu_{03}^2\mu_{50} - 3\mu_{12}^2\mu_{41} + \mu_{12}^2\mu_{03}\mu_{50})/\mu_{00}^{11} && (15)\\
I_4 &= (2\mu_{30}\mu_{12}\mu_{41}\mu_{05} - 8\mu_{30}\mu_{12}\mu_{32}\mu_{14} + 6\mu_{30}\mu_{12}\mu_{23}^2 - \mu_{30}\mu_{03}\mu_{50}\mu_{05} + 3\mu_{30}\mu_{03}\mu_{41}\mu_{14} \\
&\quad -2\mu_{30}\mu_{03}\mu_{32}\mu_{23} - 2\mu_{21}^2\mu_{41}\mu_{05} + 8\mu_{21}^2\mu_{32}\mu_{14} - 6\mu_{21}^2\mu_{23}^2 + \mu_{21}\mu_{12}\mu_{50}\mu_{05} \\
&\quad -3\mu_{21}\mu_{12}\mu_{41}\mu_{14} - 2\mu_{21}\mu_{12}\mu_{32}\mu_{23} + 2\mu_{21}\mu_{30}\mu_{50}\mu_{14} - 8\mu_{21}\mu_{03}\mu_{41}\mu_{23} \\
&\quad +6\mu_{21}\mu_{03}\mu_{32}^2 - 2\mu_{12}^2\mu_{50}\mu_{14} + 8\mu_{12}^2\mu_{41}\mu_{23} - 6\mu_{12}^2\mu_{32}^2)/\mu_{00}^{12} && (16)\\
I_5 &= (\mu_{30}\mu_{41}\mu_{23}\mu_{05} - \mu_{30}\mu_{41}\mu_{14}^2 - \mu_{30}\mu_{32}^2\mu_{05} + 2\mu_{30}\mu_{32}\mu_{23}\mu_{14} - \mu_{30}\mu_{23}^3 - \mu_{21}\mu_{50}\mu_{23}\mu_{05} \\
&\quad +\mu_{21}\mu_{50}\mu_{14}^2 + \mu_{21}\mu_{41}\mu_{32}\mu_{05} - \mu_{21}\mu_{41}\mu_{23}\mu_{14} - \mu_{21}\mu_{32}^2\mu_{14} + \mu_{21}\mu_{32}\mu_{23}^2 + \mu_{12}\mu_{50}\mu_{32}\mu_{05} \\
&\quad -\mu_{12}\mu_{50}\mu_{23}\mu_{14} - \mu_{12}\mu_{41}^2\mu_{05} + \mu_{12}\mu_{41}\mu_{32}\mu_{14} + \mu_{12}\mu_{41}\mu_{23}^2 - \mu_{12}\mu_{32}^2\mu_{23} - \mu_{03}\mu_{50}\mu_{32}\mu_{14} \\
&\quad +\mu_{03}\mu_{50}\mu_{23}^2 + \mu_{03}\mu_{41}^2\mu_{14} + 2\mu_{02}\mu_{41}\mu_{32}\mu_{23} + \mu_{03}\mu_{32}^3)/\mu_{00}^{13} && (17)
\end{aligned}
$$

Using radial coordinates the moment function from eq. 1 becomes

$$\Phi_{pq} = \int\int r\,\Psi_{pq}(r,\theta) f(r,\theta)\mathrm{d}r\mathrm{d}\theta \tag{18}$$

or may often been written (Mukundan and Ramakrishnan, 1998) as

$$\Phi_{pq} = \int\int r^{p+q+1}\,\widetilde{\Psi}_{pq}(\theta) f(r,\theta)\mathrm{d}r\mathrm{d}\theta \tag{19}$$

where $\Psi_{pq}(x,y) = r^{p+q}\,\widetilde{\Psi}_{pq}(\theta)$.

2.1.3 Zernike Moment Invariants

The Zernike polynomial (Teague, 1980) of order n and repetition L is defined as

$$V_{nL}(r,\theta) = R_{nL}(r)\exp(iL\theta) \tag{20}$$

where $R_{nL}(r)$ is a radial polynomial

$$R_{nL}(r) = \sum_{s=0}^{\lfloor 1/2(n-|L|)\rfloor} (-1)^s \frac{(n-s)!}{s!\,\lfloor 1/2(n-2s+|L|)\rfloor!\,\lfloor 1/2(n-2s-|L|)\rfloor!} r^{n-2s} \tag{21}$$

with $n = 0, 1, \ldots\ , 0 \le |L| \le n$ and $n - |L|$ must be even. The Zernike moment of order n and repetition L is defined as

$$A_{nL} = \frac{n+1}{\pi}\int_0^{2\pi}\int_0^1 rV_{nL}^*(r,\theta) f(r,\theta)\mathrm{d}r\mathrm{d}\theta \tag{22}$$

and Zernike invariants as

$$(ZMI)_{n0} = A_{n0} \tag{23}$$

$$(ZMI)_{nL} = |A_{nL}|\cdot|A_{nL}| \tag{24}$$

Secondary invariants $(ZMI)_{nz}$where $z > n$ are not further treated here.

For discretization of the Zernike moment the image coordinates must be transformed into radial and further be normalized to fit onto the unit circle (Hwang and Kim, 2006). This is done by

$$r_{ij} = \frac{\sqrt{(2x-N+1)^2+(N-1-2y)^2}}{N-1} \quad (25)$$

$$\theta_{ij} = \arctan\left(\frac{N-1-2y}{2x-N+1}\right) \quad (26)$$

Then eq. 22 becomes

$$A_{nL} = \frac{n+1}{\lambda_n} \sum_{i=0}^{N-1} \sum_{j=0}^{N-1} R_{nl}(r_{ij}) \exp(iL\theta_{ij}) f(i,j) \quad (27)$$

where λ_n is the amount of Pixel within the unit circle and N the length of the longest axis of an image object.

2.1.4 Pseudo-Zernike Moment Invariants

Similar to Zernike moments the Pseudo-Zernike polynomial (Teh and Chin, 1988) is defined as

$$\widetilde{V}_{nL}(r,\theta) = \widetilde{R}_{nL}(r) \exp(iL\theta) \quad (28)$$

where $\widetilde{R}_{nL}(r)$ is given through

$$\widetilde{R}_{nL}(r) = \sum_{s=0}^{n-|L|} (-1)^s \frac{(2n+1-s)!}{s!(n-s-|L|)!\,\lfloor 1/2(n-s+|L|+1)\rfloor!} r^{n-s} \quad (29)$$

under restrictions $n = 0, 1, \ldots\ , 0 \leq |L| \leq n$ and the moment function becomes

$$\widetilde{A}_{nL} = \frac{n+1}{\pi} \int_0^{2\pi} \int_0^1 r\widetilde{V}^*_{nL}(r,\theta) f(r,\theta) \mathrm{d}r\mathrm{d}\theta \quad (30)$$

The discretization is done like in eq. 27, substituted with $\widetilde{R}_{nL}(r)$ for the polynomial.

Pseudo-Zernike moment invariants are defined as

$$(PZMI)_{n0} = \widetilde{A}_{n0} \quad (31)$$

$$(PZMI)_{nL} = |\widetilde{A}_{nL}| \cdot |\widetilde{A}_{nL}| \quad (32)$$

Like in 2.1.3 $(PZMI)_{nz}$ where $z > n$ are not processed.

Remarks

Since ZMI and PZMI ranges between $[0, \infty)$, the logarithm may be taken for a clearer overview, thus it is optioned in the Feature editing dialog. Furthermore this does not apply to AMI and GMI which possible ranges $(-\infty, \infty)$, but it is executable as customized feature with $log(abs(MI))$.

The $(ZMI)_{nz}$ and $(PZMI)_{nz}$ with $z > n$ is given by

$$(ZMI)_{nz} = (\widetilde{A}_{nL})^* \cdot (\widetilde{A}_{mh})^p + \left((\widetilde{A}_{nL})^* \cdot (\widetilde{A}_{mh})^p\right)^* \quad (33)$$

$$(PZMI)_{nz} = (\widetilde{A}_{nL})^* \cdot (\widetilde{A}_{mh})^p + \left((\widetilde{A}_{nL})^* \cdot (\widetilde{A}_{mh})^p\right)^* \quad (34)$$

where L is divisible by h, $p \geq 1$, $p = L/h$, $2 \leq m \leq n$, $m \geq h$ and $z = n+L+p$. The decomposite of z could not be done in an automatic way, hence $z > n$ is not treated.

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